A comparison of turbulent closure models with direct numerical simulations of convection

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Introduction

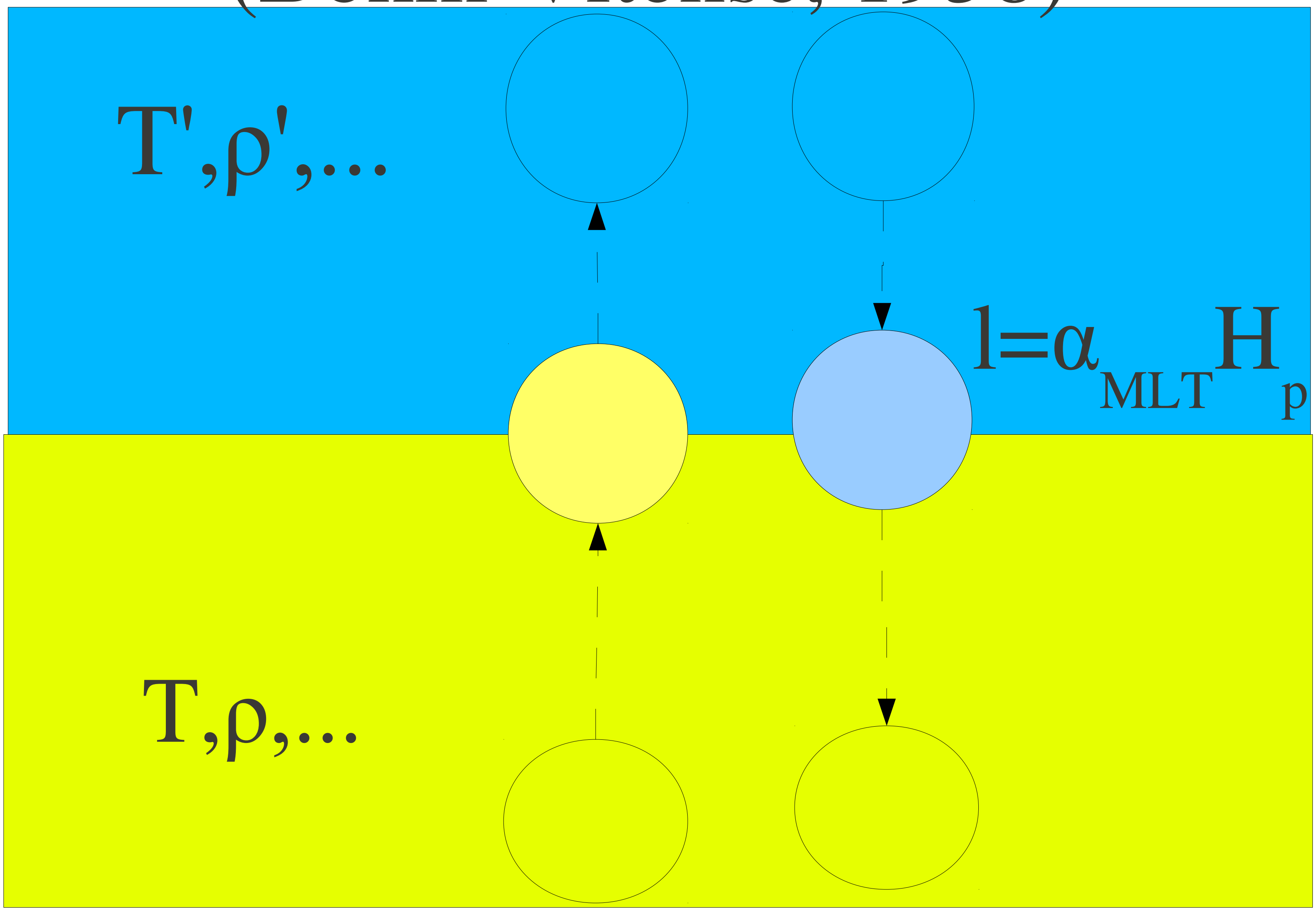
- The direct numerical simulations (DNS) of differential rotation, meridional flows and the magnetic fields of the Sun are complicated by

  -the need to model turbulent convection and

  -the wide range of relevant scales.

- Simple models such as the mixing length theory have previously been used to tackle with these problems.
- The computing power of modern computers allows us to move beyond the earlier models.

The mixing length model
(Böhm-Vitense, 1958)



Mean field models

- One way to study turbulence is to model large and small scale motions separately.

=> Reynolds decomposition:

$$x = \overline{X} + x'$$

+ The Reynolds rules:

$$\overline{\overline{X}} = \overline{X}, \quad \overline{\overline{X}_1 + \overline{X}_2} = \overline{X}_1 + \overline{X}_2, \quad \overline{\overline{X}_1\,\overline{X}_2} = \overline{X}_1\,\overline{X}_2, \quad \overline{x'\,\overline{X}} = 0, \quad \overline{x_1 x_2} = \overline{X}_1\,\overline{X}_2 + \overline{x'_1 x'_2}$$

$$\overline{\frac{\partial x}{\partial t}} = \frac{\partial \overline{X}}{\partial t}, \quad \overline{\frac{\partial x}{\partial r_i}} = \frac{\partial \overline{X}}{\partial r_i}$$

Closure problem in a nutshell

- Deriving equations of motion for mean velocity field one encounters a closure problem:

$$\frac{\partial \overline{U_i}}{\partial t} = \ldots + \frac{\partial \overline{u'_i u'_j}}{\partial r_j} = \ldots + \frac{\partial R_{ij}}{\partial r_j}$$

$$\frac{\partial \overline{R_{ij}}}{\partial t} = \ldots + \frac{\partial \overline{u'_i u'_j u'_k}}{\partial r_k} = \ldots + \frac{\partial N_{ijk}}{\partial r_k}$$

$$\vdots$$

$$\frac{\partial N_{i_1 i_2 \ldots i_n}}{\partial t} = \ldots + \frac{\partial N_{i_1 i_2 \ldots i_n + 1}}{\partial r_{n+1}}$$

- This can only be remedied by closure models, i.e. applying an approximation at some point.

Pros and cons of closure models

- Pros:

  - Analytical treatment of turbulent phenomena.

  - Computationally inexpensive.

- Cons:

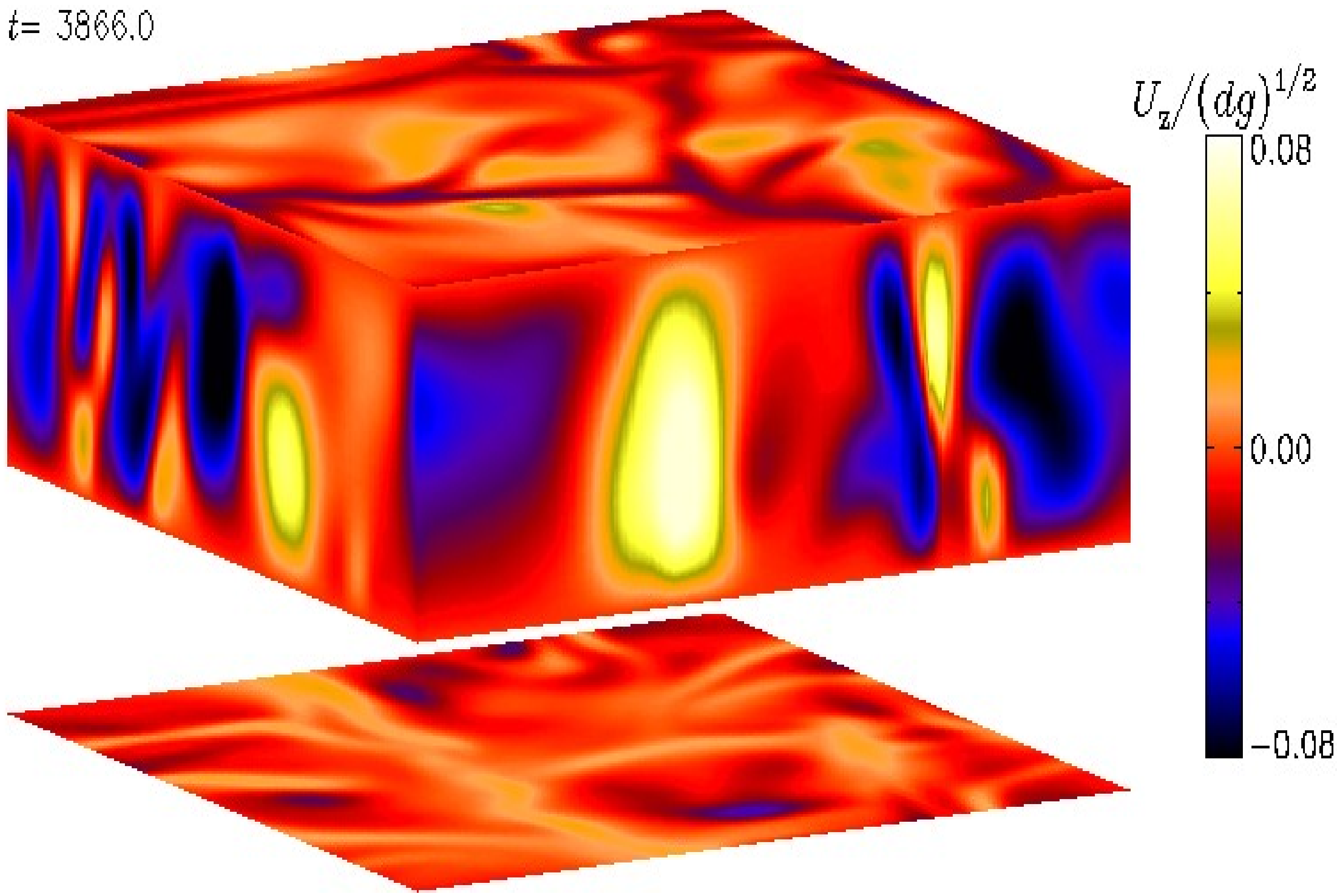
  - The assumptions used are often very drastic, and lack thorough physical justification.

  - Many closures employ several model parameters, which need need to be evaluated and explained independently.

Closure vs DNS

- Garaud et al (2010, hereafter GOMS10) proposed a closure model for turbulent convection.
- In the present work we compare the results from this closure to the DNS results from Pencil Code (http://code.google.com/p/pencil-code/).
- The DNS setup describes 3d box in the convective zone of a rotating star at colatitude θ.
- The horizontal averages from the DNS results were compared with results from 1d incompressible Boussinesq closure.

Pencil Code in action



The variables and parameters

- The mean variables employed by the GOMS10 closure are the mean velocity field $\bar{U}$ and temperature $\bar{\Theta} = T - T_0$.
- The following second order turbulent correlations are retained in the model: The Reynolds stresses, $R_{ij} = \overline{u_i' u_j'}$ the turbulent heat fluxes $F_i = \overline{\Theta' u_i'}$ and the temperature variance $Q = \overline{\Theta'^2}$.
- The physical parameters used in the model are the gravitational acceleration g, the coefficient of expansion α, the kinematic viscosity ν and thermal diffusivity χ_0.

The mean equations in the Boussinesq case

$$\dot{\overline{U}}_i = -\overline{U}_j\partial_j\overline{U}_i - \alpha\overline{\Theta}g_i - \partial_i\overline{\Psi} - 2\,\epsilon_{ijk}\Omega_j\overline{U}_k + \nu\partial_{jj}\overline{U}_i - \partial_j R_{ij} \quad (1)$$

$$\dot{\overline{\Theta}} = -\overline{U}_i\partial_i\overline{\Theta} + \chi_0\partial_{ii}\overline{\Theta} - \partial_i F_i \quad (2)$$

$$\dot{R}_{ij} + \overline{U}_k\partial_k R_{ij} + R_{ik}\partial_k\overline{U}_j + R_{jk}\partial_k\overline{U}_i + \alpha(F_i g_j + F_j g_i) - \nu\partial_{kk}R_{ij}$$
$$+2\varepsilon_{ilk}\Omega_l R_{jk} + 2\varepsilon_{jlk}\Omega_l R_{ki} = N^1_{ij}$$

$$\dot{F}_i + \overline{U}_j\partial_j F_i + R_{ij}\partial_j\overline{\Theta} + F_j\partial_j\overline{U}_i + \alpha Q g_i - \tfrac{1}{2}(\nu + \chi_0)\partial_{jj}F_i$$
$$+2\varepsilon_{ijk}\Omega_j F_k = N^2_i \quad (3)$$

$$\dot{Q} + \overline{U}_i\partial_i Q + 2F_i\partial_i Q - \chi_0\partial_{ii}Q = N^3 \quad (4)$$

The nonlinear terms replaced by GOMS10

$$N^1_{ij} = -\overline{u'_i\partial_j\Psi' + u'_j\partial_i\Psi'} - \overline{u'_i\partial_k(u'_ju'_k) + u'_j\partial_k(u'_iu'_k)} - 2\nu\overline{\partial_k u'_i\partial_k u'_j}$$

$$\to -\frac{C_1}{L}R^{1/2}R_{ij} - \frac{C_2}{L}R^{1/2}(R_{ij} - \tfrac{1}{3}R\delta_{ij}) - \nu\frac{C_\nu}{L^2}R_{ij} \quad (5)$$

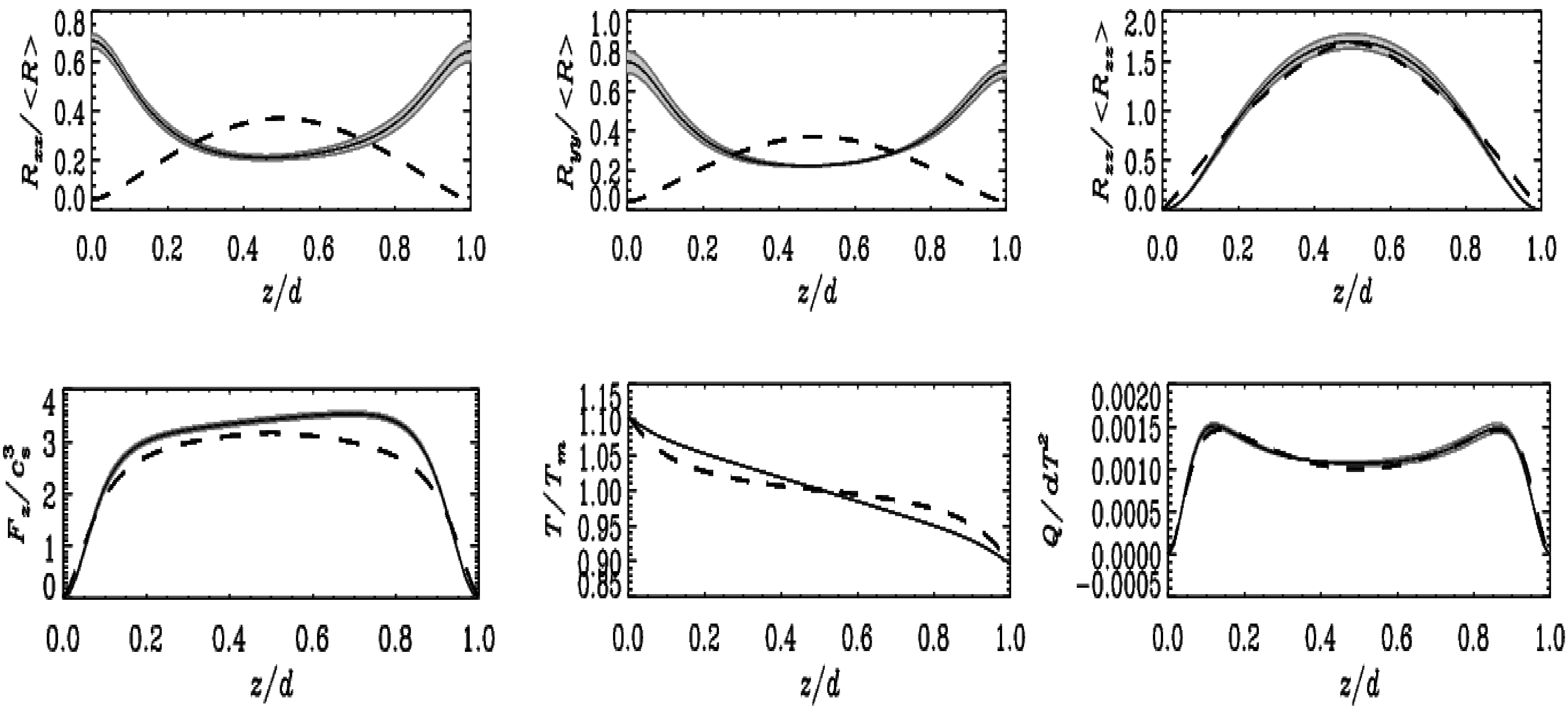
$$N^2_i = -\overline{\Theta'\partial_i\Psi'} - \overline{\Theta'\partial_k(u'_ju'_k) + u'_k\partial_k(u'_i\Theta')} + \frac{1}{2}(\nu - \chi_0)\overline{\partial_k(\Theta'\partial_k u'_i - u'_i\partial_k\Theta')}$$

$$-(\nu + \chi_0)\overline{\partial_k\Theta'\partial_k u'_i} \to -\frac{C_6}{L}R^{1/2}F_i - \tfrac{1}{2}(\nu + \chi_0)\frac{C_{\nu\chi_0}}{L^2}F_i \quad (6)$$

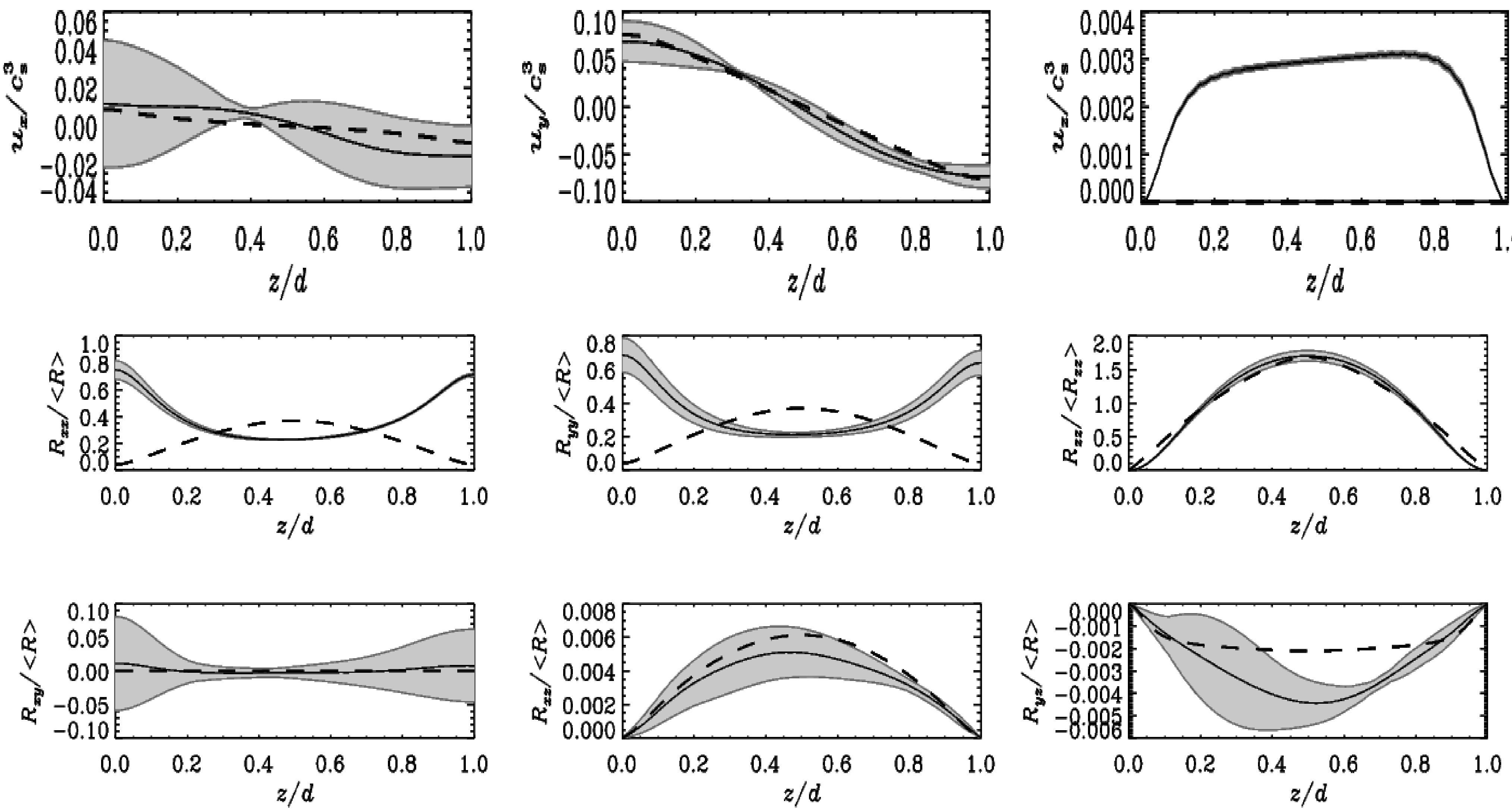
$$N^3 = -2\nu\overline{\Theta'\partial_k F_k} - 2\chi_0\overline{(\partial_k\Theta')^2} \to -\frac{C_7}{L}R^{1/2}Q - \chi_0\frac{C_{\chi_0}}{L^2}Q \quad (7)$$

Relaxation Return to isotropy
Turbulent diffusion

Slow rotation, θ=0^0 (Snellman et al 2012, in preparation)



Slow rotation, $\theta=45^0$, (Snellman et al 2012, in preparation)



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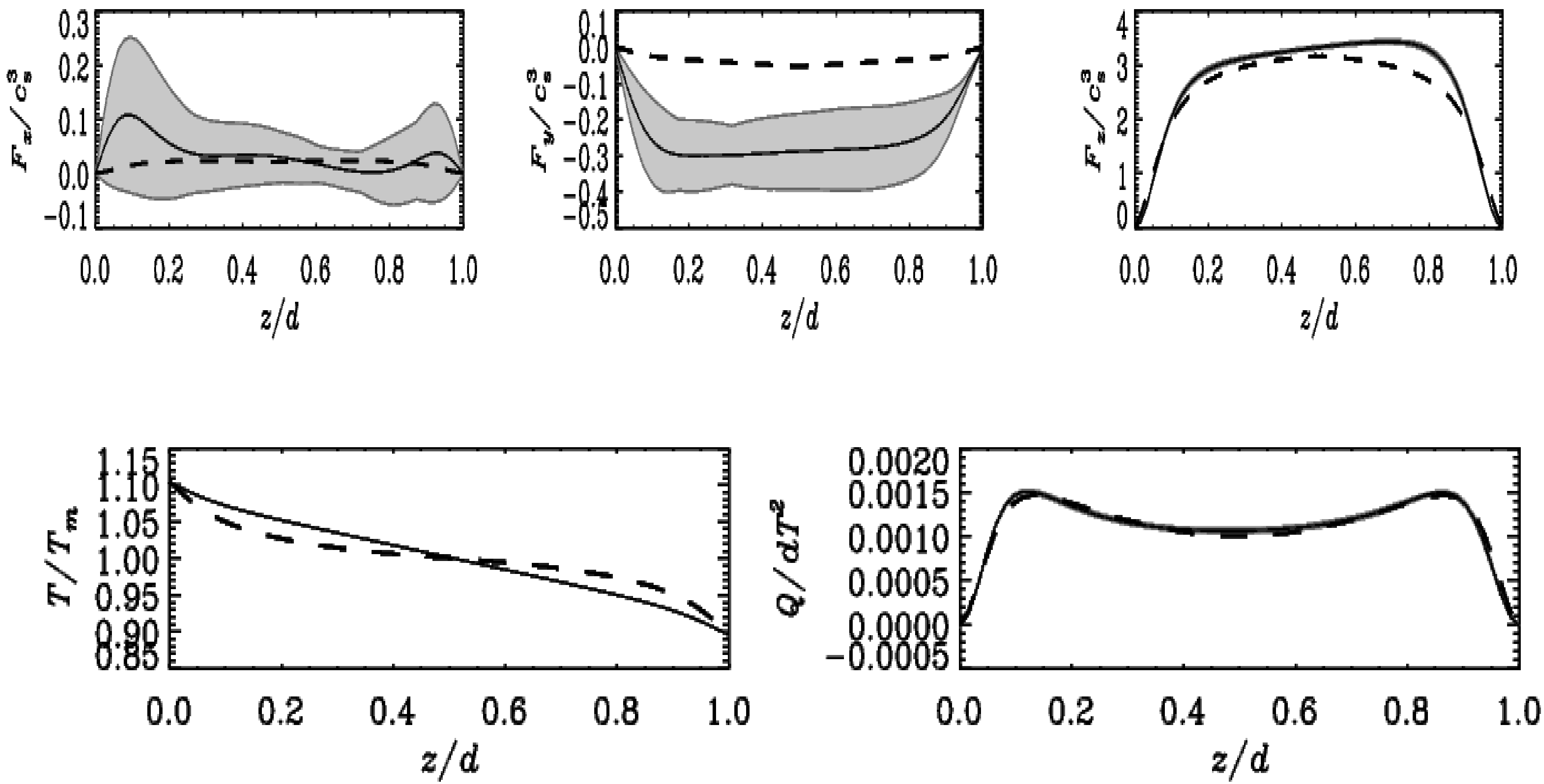
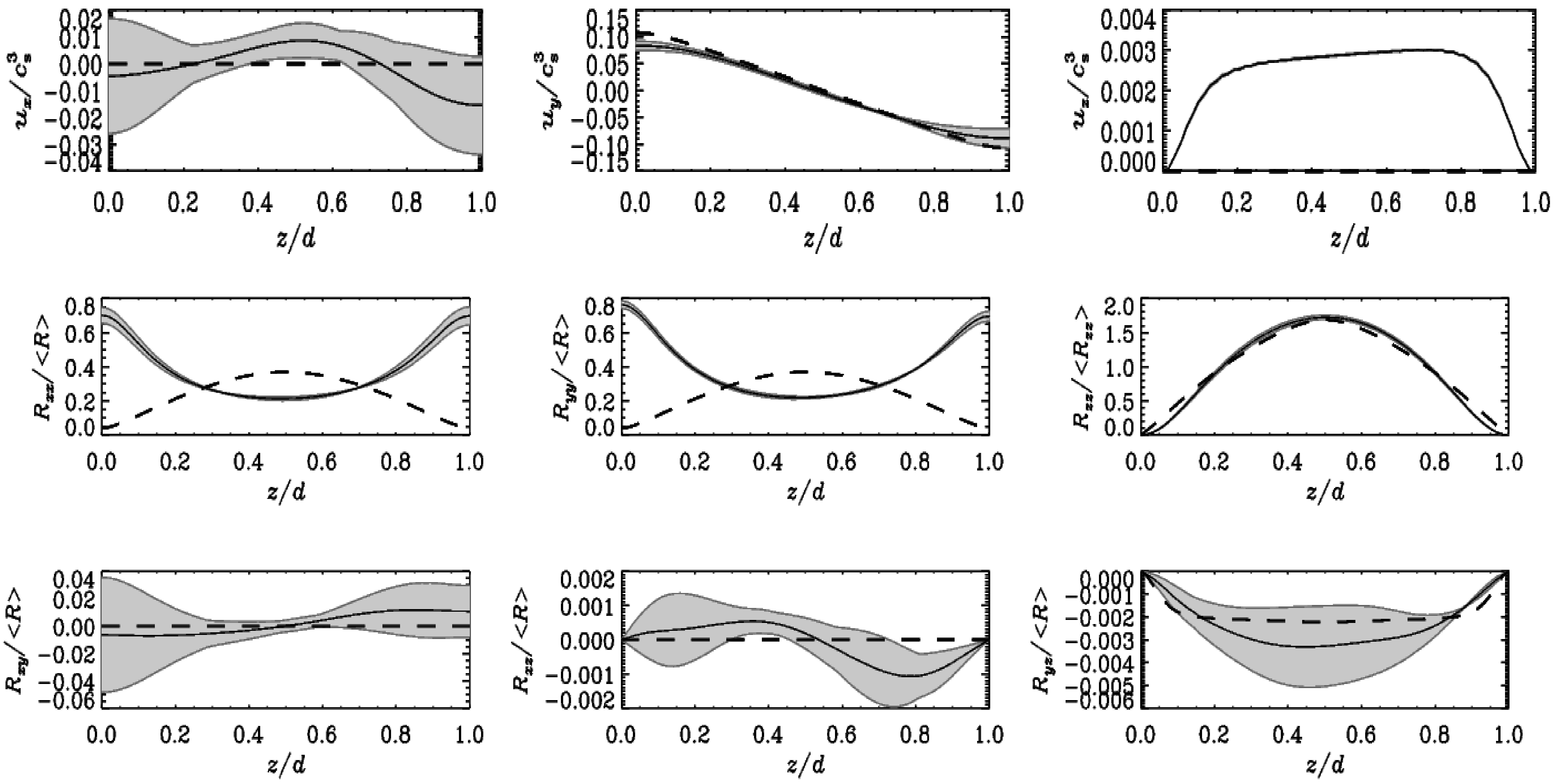


Slow rotation, $\theta=90^0$, (Snellman et al 2012, in preparation)



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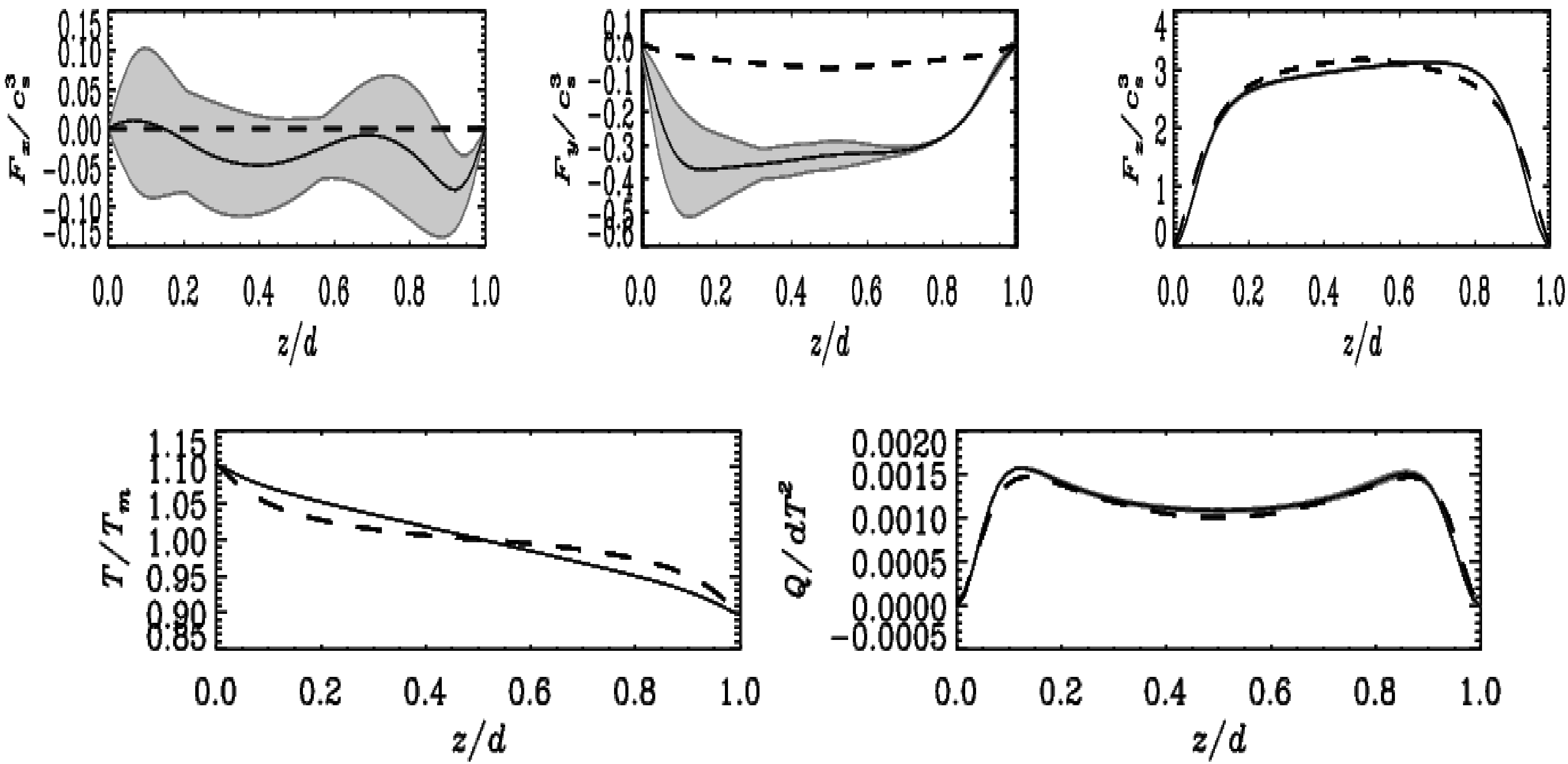

Conclusions and outlook

- In 1d application of the GOMS10 model the most striking difference to the DNS results is seen at the boundaries in R_{xx} and R_{yy}, but otherwise the results do not seem too different.
- Likely reason for this discrepancy are the coherent flow patterns in DNS that are not taken into account in the closure.
- Coming up: Boussinesq module for Pencil Code, comparisons with 0-dimensional version of the closure.

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